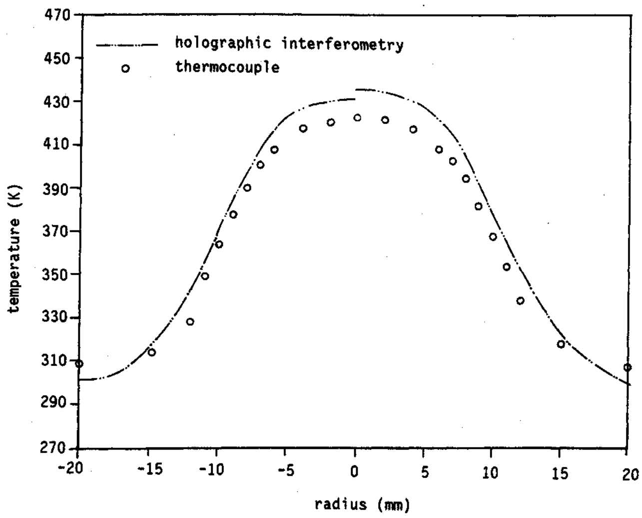

Fig. 3 Comparison of radical distributions of mean interferometric and thermocouple temperatures at $z = 20$ mm for case 2.

the inlet temperature of the heated air for case 2 is 423 K. As shown in Fig. 3, it is clear that the holographic interferometry overmeasures the mean temperatures, while the thermocouple measurements are quite consistent in the core region. The overestimations of the mean interferometric temperatures are attributed to the errors made in fringe interpolation/extrapolation. In case 1, there existed four dark fringes in the fringe pattern of each half-section and one dark fringe was located at the axis center (fringe order number to this dark fringe is $-3\frac{1}{2}$ as indicated in Fig. 2a). Therefore, the fringe number distribution in the core region can be interpolated by two neighboring fringes, i.e., $-2\frac{1}{2}$ and $-3\frac{1}{2}$. Consequently, the calculated mean interferometric temperatures agree well with the ones measured by the thermocouple in this region as shown in Fig. 2b. The nearly uniform mean interferometric temperature distribution in the core region for case 2 leads to a situation in which no dark fringe was formed around the axis center in the fringe pattern. This means that the fringe number distribution around the axis center was extrapolated from the data located in the outer flow region where the mean temperature gradients are much larger than those in the core region. This can explain why the mean interferometric temperatures in the core region were overestimated for case 2.

In summary, error analysis showed that the resolution limitation in the present image processing may cause as large as 8.6% and 8.3% uncertainties in the calculations of mean interferometric temperatures for the cases of plume-like and jet-like flows, respectively. These errors would be effectively reduced, provided a higher resolution capability of the image processing was employed in the experiment. The requirement of axisymmetry for the interest domain restricts these diagnostics applicable to measurements of mean temperature only. Nevertheless, our group has been developing a holographic tomography technique[9] capable of reconstructing a three-dimensional temperature distribution. It is believed that the instantaneously asymmetrical temperature distribution can be obtained by using such a holographic tomography technique. Another difficulty in the application of these diagnostics to thermal fields associated with high temperature gradient is due to ray bending of laser light while traversing these fields, and it remains to be studied further.

Acknowledgment

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Convective Flows in Enclosures with Vertical Temperature or Concentration Gradients

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Introduction

IN some crystal-growth techniques, the completely confined fluid phase is subject to vertical temperature and/or concentration gradients.[1,2] To gain insights on such flows, an experimental program was initiated to study flows in rectangular enclosures with vertical temperature or concentration gradients between horizontal end walls.

The present work for purely thermal cases is intended to give a comprehensive understanding of natural convection heat transfer in enclosures with thermally insulated vertical walls by systematically varying the parameters *Ra* (Rayleigh number) and *Ar* (aspect ratio). The fluid is tap water in these experiments.

An electrochemical system based on a diffusion-controlled electrode reaction is employed to create the vertical concentra-

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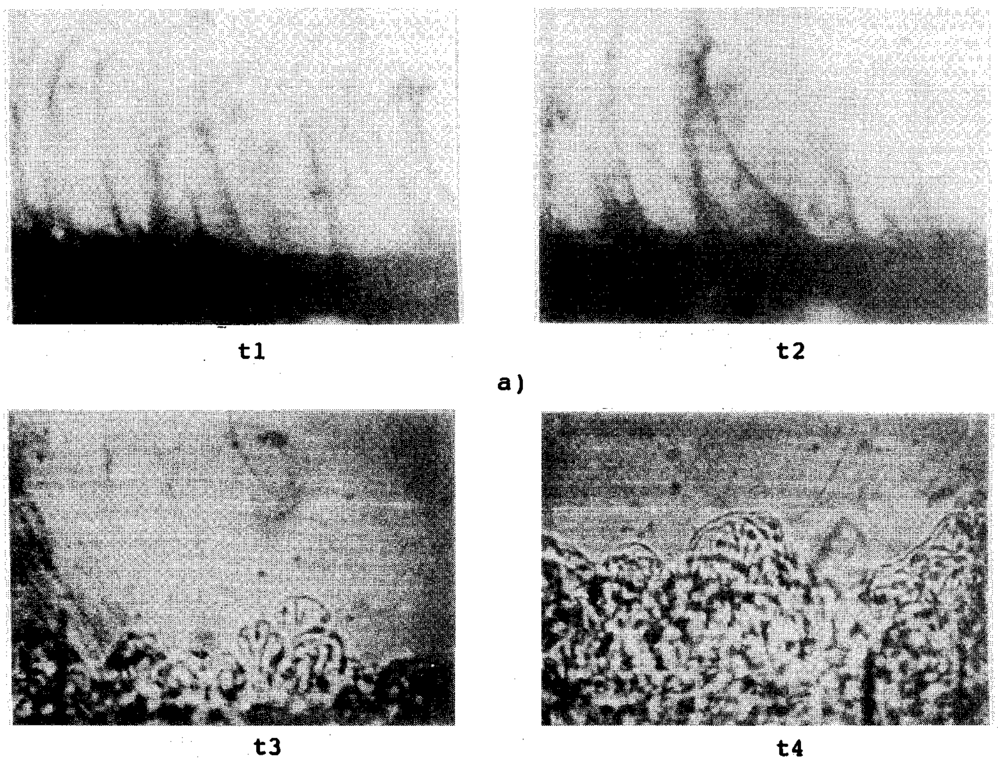

Fig. 1 Flow patterns near a) the heated lower plate (purely thermal case) with $Ar = 0.65$, $Ra = 1.5 \times 10^7$, $t2 > t1$, and $t2 - t1 = 5$ s; and b) the cathode lower plate (purely solutal case) with $Ar = 1.02$, $Ra_m = 2.08 \times 10^{11}$, $t4 > t3$, and $t4 - t3 = 5$ s.

tion gradients. The apparatus is a rectangular cavity with the horizontal end walls being the electrodes. All other walls are insulated. The fluid is a copper sulphate ($CuSO_4$)-acid (H_2SO_4) solution for purely solutal experiments. The present paper is partially based on the work by Wang and Sun.[3]

Experimental Design

With the present system, the ranges of the dimensionless parameters covered for purely thermal cases are (Prandtl number) $Pr = 4.0$ to 7.0, (Grashof number) $Gr = 1.3 \times 10^6$ to 5.0×10^8, and $Ar = 0.65$ to 2.6; and for purely solutal cases are (Schmidt number) $Sc = 2000$ to 2100, $Gr_m = 5.0 \times 10^5$ to 1.2×10^8, and $Ar = 0.68$ to 1.34.

Test Apparatus and Procedure

The test cell is a rectangular enclosure formed with four insulating Plexiglas plates and two copper plates. The width L is 7.7 cm or 8.2 cm and the height is variable so that a range of aspect ratios can be covered. The depth of the enclosure is relatively large (23.9 cm or 21.2 cm). The two copper horizontal end walls are 0.7 cm thick.

For the purely thermal cases, an electrical heating mat is bonded to the back of the bottom copper plate and a circulating water system cools the top copper plate, so that a uniform temperature ($\pm 0.05^\circ$C) can be imposed on the top surface. The method used to visualize the flow patterns for purely thermal cases is an electrochemical method called the pH-indicator method.[4] Usually, it took about 10 h to reach a quasisteady state after the heater was turned on. The experimental error in the value of (Nusselt number) Nu was estimated to be $\pm 8\%$.

The test fluid is a copper sulfate-sulfuric acid solution ($CuSO_4 + H_2SO_4 + H_2O$) for the purely solutal cases. When a voltage is applied to the electrodes, copper dissolves into the solution at the anode and is deposited at the cathode. As a result, the density of the fluid near the cathode (anode) becomes lower (higher) than that of the bulk of the solution. The migration of cupric ions in the electric field is eliminated by adding sulphuric acid to the solution, which acts as a supporting electrolyte, and thus the transport of the cupric ions is controlled only by diffusion and convection. The equipment for measuring the current and potential is the same as Refs. 2 and 3. One relatively simple way to set the concentration level at the cathode in the present system is to adjust the cell potential in such a way that the saturation (limiting) current is obtained. Under the limiting current condition, the ion concentration at the cathode surface is zero.

The cell voltage is increased stepwise by manual control, and the corresponding total current is read after each quasisteady state is attained. This procedure is repeated until a potential-current plateau is obtained. The potential stepping rate is chosen as 20 mV/40 s.

Experimental Results

Purely Thermal Cases

The flow structures were studied by the aforementioned pH-indicator technique. Based on these observations, the flow structures are shown in Fig. 1a. "Thermals" rising from the bottom of the test cell and falling from the top of the test cell are evident. The dominant motion in the test cell is that of the thermals.[5,6] To record temperature disturbances induced by the thermals, the thermocouple probe was placed as near as possible to the bottom and top surfaces of the test cell. Time-series records of the thermocouple output are shown in Fig. 2. According to Fig. 2, the amplitudes of the temperature oscillations increase near the top and bottom plates. The flow becomes supercritical. At such a Rayleigh-number level, it is known that the enclosure flow bifurcates to a time-dependent flow.[7-9] It is generally accepted that the flow is fully turbulent for Rayleigh numbers greater than about 10^5 for water ($Pr \approx 7$).

The Nusselt number data for water ($Pr = 5.5$ to 7.0) for various values of Ar are presented in Fig. 3a. The predictions of Nu by others[6,10] are also shown in this figure for comparison. Nu was found to be proportional to $Ra^{0.322}$ as seen in Fig. 3a. The value of the exponent ($n = 0.322$) points to a heat transfer mechanism by turbulent eddies. However, there appears to be a small but noticeable difference between the present data ($Hr \equiv L/W = 0.322$) and others' data ($Hr = 1$). Nu seems to increase slightly with decreasing Hr.

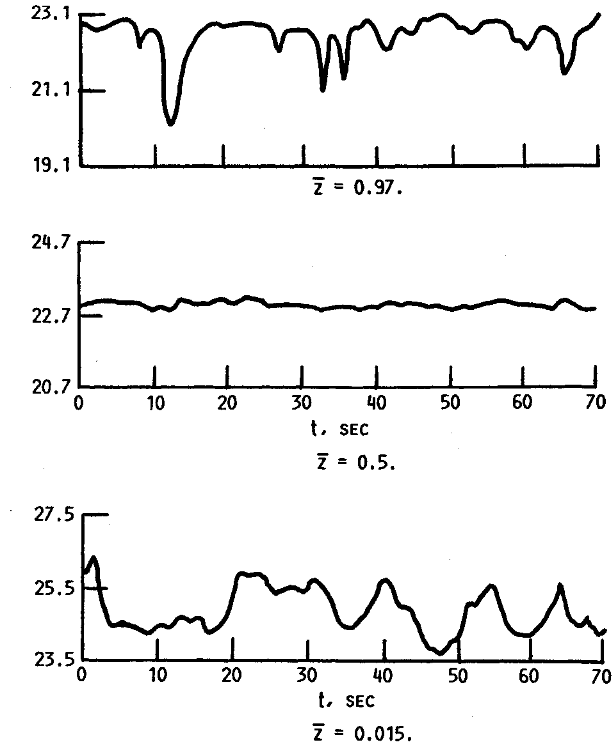

Fig. 2 Instantaneous temperature profiles at $X = 0.78$, $Y = 0.59$ for purely thermal case with $Ar = 1.30$, $Ra = 1.35 \times 10^8$, and $Th = 27.8°C$, where Th is the temperature of the bottom plate.

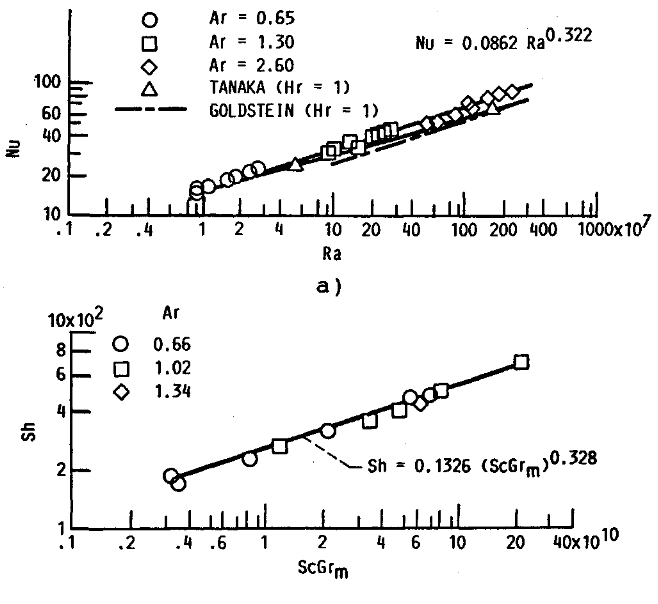

Fig. 3 Correlation of a) Nu for purely thermal case with $Hr = 0.322$ (present case); and b) Sh for purely solutal case.

Purely Solutal Cases

In the present electrochemical system operated under isothermal conditions, the deposition of cupric ions on the cathode wall leaves behind less dense fluid. The lighter fluid rises toward the top of the test section. The downward flow in the boundary layer near the anode feeds heavier fluid toward the bottom wall.[11] Simple shadowgraph studies (Fig. 1b) suggested a very complex flow structure over the $ScGr_m$ range of 1×10^9 - $2.4 \times 10.^{11}$

In the present electrochemical system, the diffusion coefficient is very small so a long time is required to obtain a truly solutal convection. Thus, the mass-transfer rate measured herein is not a steady-state value. However, the solutal boundary layers should be established on a time scale of δ_s/D, which is on the order of 10 s and is small compared to the time at which the limiting current is measured. The measured mass-transfer rate is not expected to differ much from the steady-state value. The experimental error in Sh (Sherwood number) is estimated to be $\pm 10\%$. A correlation for Sh, as a function of solutal Rayleigh number ($ScGr_m$), is presented in Fig. 3b.

Summary of Results

The results of the present experiments indicate that for the range of aspect ratios and Grashof (or Rayleigh) numbers covered,

1) The results for the purely thermal cases are adequately correlated by the equation $Nu = 0.862\ Ra^{0.322}$ in the range $9 \times 10^6 < Ra < 2 \times 10^9$.

2) Temperature oscillations and the exponent 0.322 found in the purely thermal cases reflect a high degree of flow instability in the present work.

3) For the purely solutal cases, the mass-transfer data can be correlated in the equation $Sh = 0.1326\ (ScGr_m)^{0.328}$ in the range $1 \times 10^9 < Ra_m < 2.4 \times 10^{11}$.

Very recently, the results of a similar study from Goldstein et al. give $Sh = 0.0659(ScGr_m)^{1/3}$ for $3 \times 10^9 < Ra_m < 5 \times 10^{12}$ (Ref. 9).

Figure 3 shows clearly that all results are well-correlated by the equation $Nu = C_1 Ra^n$ and $Sh = C_2 Ra_m^n$. From a practical point of view, $n = 1/3$ is acceptable as a regression parameter, at least within the experimental Ra (or Ra_m) range.

In the present work, comparison of the new correlations of mass-transfer data to the results of heat-transfer investigations could serve to give us some enlightenment about the mechanism of the transport process under discussion.

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